

Computational Study of Unsteady Diffracted Shock Wave over Convex Sharp Edges

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Abstract—Shock diffraction occurs when a moving normal shock wave undergoes a sudden area of expansion. Capturing the blast wave is very important as it has fracturing effect on solid bodies and it creates small-scale debris and dust. Blast waves are generated from detonations and it involves high-pressure-ratio shock waves moving at high velocity. This paper presents a numerical simulation of solving Euler equations to capture unsteady shock wave diffraction over 90° step corner. The contact surface, shock and expansion waves are very well produced and validated. The created moving shock wave is made to be diffracted over 30°, 60°, 90°, 120°, 150° steps to run for a short interval of time of unsteady flow at different shock Mach numbers ($M_s=1.65$ to 3.0). The changes of flow characteristics with the increase of shock Mach number are reported here. Euler computations produce flow separation near to the diffracted edge. A good resolution of the perturbed region is identified behind the diffracted shock wave. The secondary shock, vortex core, slipstream, terminator are very well produced. Threshold limiting value of acoustic expansion wave to flow into upstream is also identified.

Keywords: Shock diffraction, expansion wave, supersonic flow, blast wave, Euler computations, slipstream, secondary shock, vortex core.

1. INTRODUCTION

Shock diffraction at sharp edges is an important feature in many gas dynamic problems of interest and is even significant for the prediction of blast wave's interaction with structures in defense research. Detonations or explosions create blast waves including high - pressure ratio shock waves moving at high speeds. Capturing the blast wave is necessary as it is fracturing effect on solid bodies. The blast wave front resulting from an explosion is typically spherical, but small segments of the shock wave in the far field can be modeled as planar to study shock diffraction over obstacles. In these problems, obstacles include mostly manmade structures and vehicles. Such targets can be ideally taken as primitive geometric shapes such as rectangles, wedges and circles.

Here, the main focus is to study the perturbed region behind the diffracted shock wave. By taking the help of the literature

survey, it is found that the Sod problem [1] is an essentially one-dimensional flow discontinuity problem which provides a good test of a compressible code's ability to capture moving shocks and contact discontinuities with a small number of zones and to produce the correct density profile. Wide variety of flow characteristics occur in the perturbed region behind the diffracting shock and have been described in detail [2]. A new flow peculiarity at the corner had been introduced due to Mach numbers in the range of 1.6-1.87 and it was supposed to be the tail of the Prandtl-Meyer fan called terminator [2]. A number of experimental results for shock diffraction over a convex corner have been published. Numerical results are in abundance [3] likewise. With the formation of a series of small vortices due to the numerical solution of the compressible Euler equations, the shear layer becomes unstable and this had not been observed in shock tube experiment [4]. They explored the use of the Navier-Stokes equations and found that additional dissipation by the application of a turbulence model was needed to imitate the experimental results. Analytical investigations have been carried out by skews [5] for studying the point of intersection between the incident wave and the reflected acoustic wave and it has been extensively studied using Whitman's theory [7].

The fact that flux-vector splitting scheme is based on characteristic decomposition of the convective fluxes; has been proposed by Van Leer [8]. It performs quite well in the case of Euler equations. Roe [9] has performed a comparative study of the upwind schemes developed earlier, classified into flux-vector and flux-difference splitting, and has pointed out their successes and failures. A new flux splitting formula has been established by Liou and Steffen [10], namely AUSM. The AUSM resulted in correct solution in the blunt body problem without difficulty in every test in a wide variation of flow condition and grids, where the Roe splitting failed. Liou [11] again introduced modified form of AUSM i.e. AUSM+. In addition to the described accuracy and reliability, the AUSM+ requires little computational effort only linearly

proportional to the number of equation considered. A study by Meadows et al. [12] used a second order upwind finite volume scheme for solving the two-dimensional Euler equations. Shocks were captured using this method in spite of any numerical oscillations. In the 18th International Symposium on shock waves, Takayama & Inoue [13] clearly explained that numerical simulations can represent very well the diffracting shock wave, expansion waves and the main vortex. Several investigations by Hanel et al. [15], carried out with the Navier-Stokes equations divulged that splitting errors in the momentum and the energy equations smear out the boundary layers and also lead to inaccurate stagnation and wall temperatures. Hanel and Schwane [16] suggested a modification to the momentum flux in the direction normal to the boundary layer.

Origin of compression attached to the shock front was unknown and theories at that time predicted a wave with perfect anti symmetry (Ribner, [17]). Experiments and numerical results have shown that the shock compresses an initially circular vortex into an elliptical one. The acoustic pressure field formed due to shock-vortex interaction has been predicted by Ribner [18]. This shows good agreement with experiments of Dosanjh and Weeks [19] except very close to the shock front. Ellzey et al. [20] explained the significance of shock distortion in forming acoustic wave for a strong interaction.

Present unsteady flow simulation is carried over 30° - 150° step angles in the multiple of 30° with shock Mach numbers ($M_s=1.65, 2.0, 2.5, 3.0$).

Proper understanding of the flow physics and building precise numerical models have become necessary. Most previous studies commonly considered a single sharp edge connecting two plane surfaces, as shown in Fig. 1 along with a plane approach shock.

A number of flow features can be noted from a variety of experimental studies done before. At relatively small diffraction angles (less than 20° , Fig. 1a) the flow remains attached after crossing the diffraction edge. A secondary rearward-facing shock wave (stagnation wave) is formed which matches the expanded flow downstream to the flow behind the diffracted shock wave. At Fig. 1b, flow separation is geared up by growing strength of secondary wave at a large diffraction angle. The flow separation position moves nearby the diffraction edge as we increase the angle in Fig. 1c. Thus the separated flows at smaller diffraction angles are expected to be viscosity-dependent. We would hope that Euler computations would provide a reasonable flow model even after reaching "sharp-edged" separation.

Figure 1c depicts the incident plane wave (A), the diffracted wave (B) and the front of the reflected expansion wave (C) which travels back into the post-shock region and is the

demarcation between uniform and non-uniform flow. Here, the acoustic wave does not penetrate upstream of diffraction edge because the induced post-shock flow is supersonic, and the upstream part (C) is leading Mach line of a Prandtl-Meyer expansion wave. The gas is accelerated and turned parallel to the separation streamline (D). The rearward-facing shock wave (E) shocks the flow which is matching the expansion to post-diffraction shock.

The slipstream rolls up into a vortex (F) and thereby interacts with the wave. The part of flow processed by diffracting shock from that processed by incident wave is separated by another observable phenomenon, which is called the contact surface/vortex sheet (G). An overshoot is first being developed with a reflexive contour near diffraction wall for relatively strong shock waves (Fig. 1c). In some cases, the necessary deceleration of the reverse flow is achieved by the third shock (H).

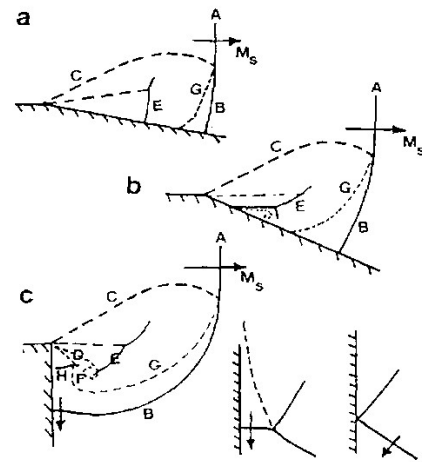


Figure 1a-c. Schematic of diffraction at a convex edge, given by Hillier [3]. a. Small diffraction angle with attached flow, b. large angle with separation downstream of the edge, c. diffraction at a 90° edge. In each case the post incident shock flow is supersonic so that there is no upstream influence.

2. METHODOLOGY

The moving shock wave produced, is made to move over 90° step corner.

The governing Euler equations describe conservation laws for mass, momentum. Those relations along with ideal gas state and internal energy equations for a compressible flow are shown below respectively.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u V) = -\frac{\partial p}{\partial x} \quad (2)$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v V) = -\frac{\partial p}{\partial y} \quad (3)$$

$$p = \rho RT \quad (4)$$

$$\rho e = \frac{p}{\gamma - 1} \quad (5)$$

In case of moving normal shock waves, gas behind the wave is being dragged by the wave. All the velocities are calculated relative to the shock propagation.

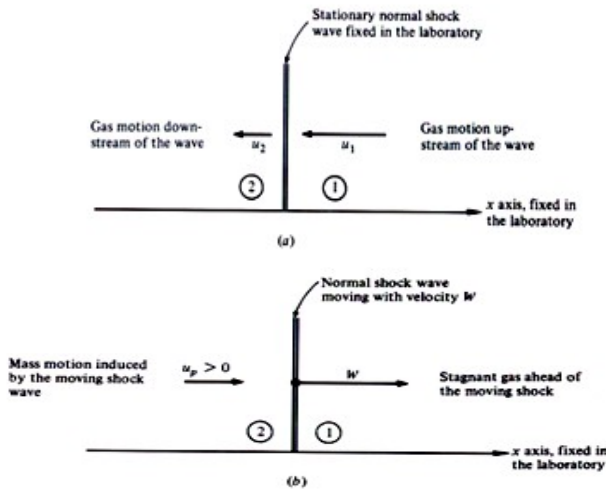


Figure 2. Schematic of stationary and moving shock waves.

W=Shock wave velocity,

u_p =Mass motion of induced gas behind moving shock,

p_2 =Upstream pressure,

p_1 =Downstream pressure.

A. Numerical details of Shockwave diffraction experiment.

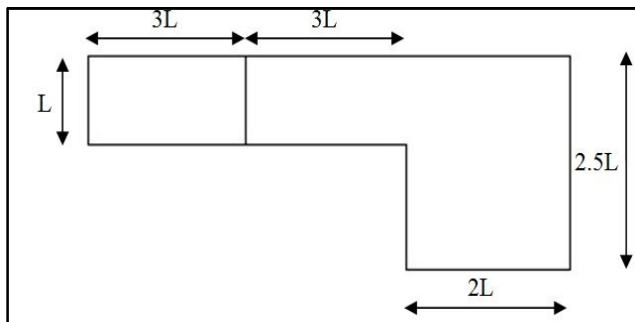


Figure 3. Computational Flow Domain.

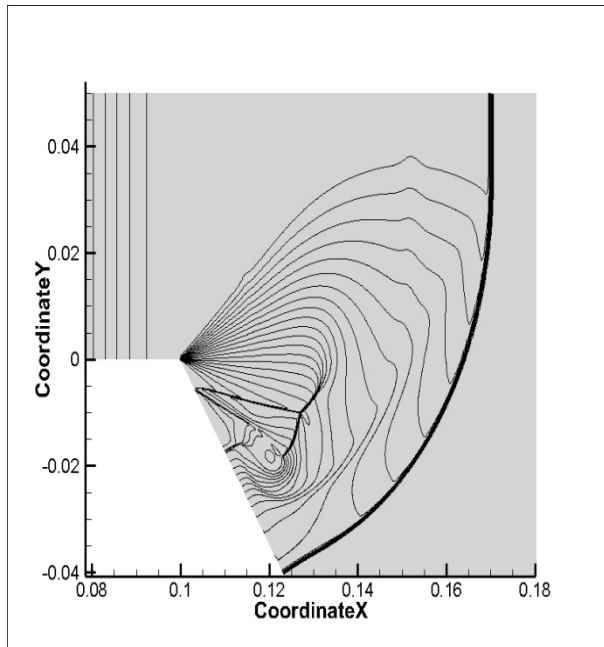
Final computations are performed for the spatial domain $0 \leq x \leq 8L$ shown in Fig. 3 of 90° step corner. Very fine meshing with about 200000 elements is considered. The two-dimensional time-dependent Euler equations coupled with the equation of state (4) and internal energy (5) are numerically solved. The Euler equations are solved by the implicit finite volume formulation using structured quadrilateral cells that covers the whole computational domain. The solver contains upwind biased schemes for calculating the AUSM flux and the scheme is second-order accurate in time and space. Obtained density contours are listed in Fig.4 and Fig.5.

3. RESULTS AND DISCUSSION

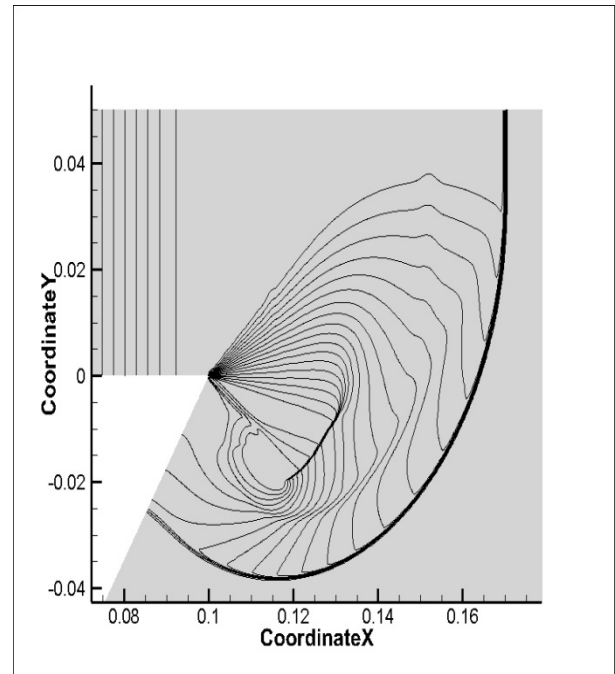
The position of the shockwave is also calculated with a simple mathematical statement and a time size is maintained. As the shockwave is followed by contact discontinuity, a certain time size has to be maintained so that contact discontinuity must not reach near to the step corner, thus the flow characteristics will be not be disturbed behind the diffracted shockwave.

Computations have been performed for a perfect gas with $\gamma=1.4$ on 200000 mesh elements with incident shock Mach numbers ranging from 1.65-3.0 over a 90° step corner at first. Then it has been extended to other step angles. After a time size is reached, computations are terminated so that the exact location of the shock wave is captured and hence it lay to be stationary at that location. This time size is maintained to make the contact discontinuity not to reach near to the step corner. A regular shock reflection occurs followed by Mach-reflection after the diffraction of the incident shock wave. Thus the reflected wave is propagating slightly upstream of the edge for these subsonic post-shock flows which can be observed from the density contours shown in subsequent figures. The angle made by terminator and slipstream are found to be decreasing through the density contours upon an increase in values of M_s . A supersonic flow can be clearly visible behind secondary shock through Mach number contours. It is enclosed between the Prandtl-Meyer fan expansion lines referred as terminator, slipstream with the secondary shock.

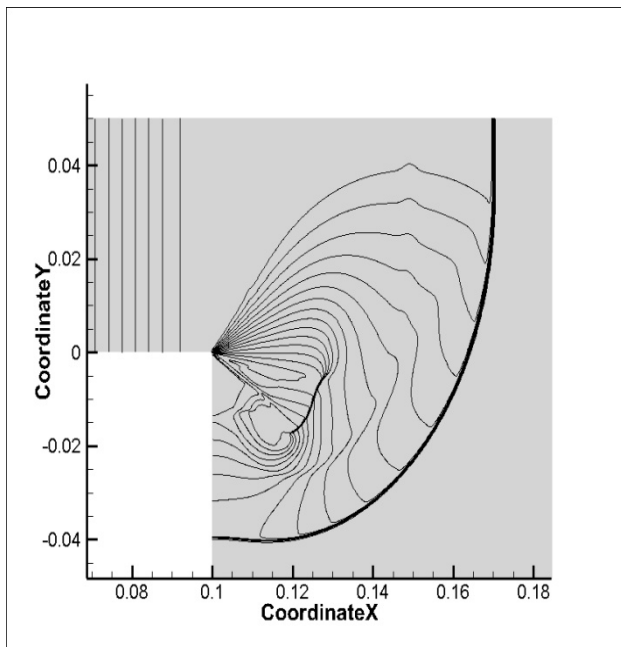
The diffracted shock wave, reflected acoustic waves/expansion waves, slipstream, vortex, contact surface/vortex sheet, secondary shocks are all captured with sufficient mesh refinement for $30^\circ, 60^\circ, 90^\circ, 120^\circ$ and 150° steps with Mach No.s-1.65, 2, 2.5, 3. These are all validated with existing experimental & numerical well established results. Also some grid independence test was carried out. Figure 4 and figure 5 represents the shadowgraph images for $60^\circ, 90^\circ, 120^\circ$ and 150° steps altogether. The slipstream remains attached to the wall in all the cases.



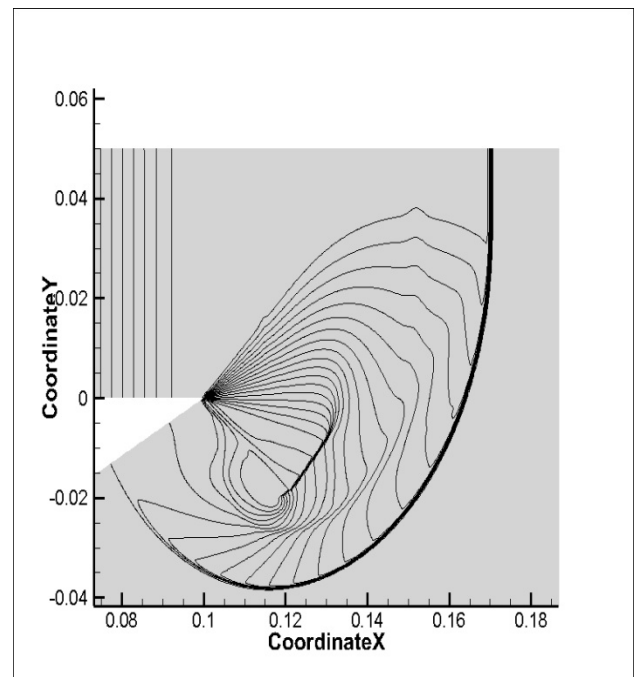
60° step, Mach No.-3



120° step, Mach No.-3



90° step, Mach No.-2.5



150° step, Mach No.-3

Figure 4: Density Contours for Different step angles (60° & 90°)

Figure 5: Density Contours for Different step angles (120° & 150°)

At higher Mach numbers, the secondary shock is beginning to appear as visible one.

With an obtuse angle, the slipstream doesn't propagate further with increase in Mach number. The circulation or vortex strength is increased significantly.

At Mach No.-3, the vortex almost touches the wedge tip. Secondary shock also becomes more prominent.

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